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Mathematics, practicality and social segregation. Effects of an overtly stratifying school system

Matemáticas, la Práctica y la segregación social. Efectos de un sistema escolar estratificado

Matemáticas, a prática e a segregação social. Efeitos de um sistema escolar estratificado

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This article aims at investigating the form of mathematical knowledge that is transmitted in a secondary school in a context of social and institutional segregation. This article follows the premise that schools are actors in the differential distribution of different forms of knowledge to different social groups. Based on Bernstein's sociology of education, this article develops an analysis of classroom interaction and discusses it within the frames of the organisation of the school system and its socio-political relations. This methodology allows for making visible how the structuring of mathematical instruction affects the social regulation of students. The analysis reveals that the pedagogic practice is likely to lead students into a pedagogic cul-de-sac: students are deprived any form of mathematical consciousness, in esoteric as well as in mundane form. The conclusion suggests the importance of transmitting specialised knowledge in relation to its internal structure, also in contexts of intense segregation.

Keywords: Sociology of education, Secondary Education, Mathematics, Knowledge, Segregation, Recontextualisation.

Este artículo tiene como objetivo investigar cómo se transmite el conocimiento matemático en una escuela secundaria en un contexto de segregación social e institucional. Este artículo sigue la premisa de que las escuelas distribuyen diferencialmente el conocimiento para los diferentes grupos sociales. Partiendo de la sociología de la Educación de Bernstein, este artículo desarrolla un análisis de la interacción dentro del aula, los marcos de organización del sistema escolar y sus relaciones socio-políticas. Esta metodología permite hacer visible cómo la estructuración de la instrucción matemática afecta a la regulación social de los estudiantes. El análisis revela que la práctica pedagógica priva a los estudiantes de cualquier forma de conciencia matemática, en la esotérica y en forma mundana. Las conclusiones sugieren la importancia de la transmisión de conocimientos especializados en relación con su estructura interna, también en contextos de intensa segregación.

Descriptores: Sociología de la educación, Educación Secundaria, Conocimiento, Segregación , Recontextualización

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Este artigo tem como objetivo investigar como se transmite o conhecimento matemático em uma escola secundária cujo contexto é de segregação social e institucional. Este artigo segue a premissa de que as escolas distribuem diferencialmente o conhecimento para os distintos grupos sociais. Partindo da sociologia da Educação de Bernstein, este artigo desenvolve uma análise da interação em classe, os marcos de organização do sistema escolar e suas relações sócio-políticas. Esta metodologia permite tornar visível como a estruturação da instrução matemática afeta a regulação social dos estudantes. A análise revela que a prática pedagógica priva os estudantes de qualquer forma de consciência matemática, na esotérica e na forma mundana. As conclusões sugerem a importância da transmissão de conhecimentos especializados em relação com sua estrutura interna, também em contextos de intensa segregação.

Descritores: Sociologia da educação, Escola Secundária, Matemática, Conhecimento, Segregação, Recontextualização.

Introduction

Schools are supposed to be places that open up opportunities for students. In schools students should acquire the skills, competencies and knowledge that societies have defined as being beneficial and foundational in order to become a full member of society. However, the fact that schools do not perform this function equally well for all social classes has been known for decades (e.g. Anyon, 1981; Bourdieu & Passeron, 1977; Bowles & Gintis, 1976; Willis 1977) and has been subject to research since. Educational studies have recognized the problem of social reproduction as a problem of school while certain subjects such as literacy or mathematics have been given particular attention as being especially relevant to social reproduction. Since the 'democratization of mathematics education' in the nineteen-eighties (Valero, 2013), mathematics education has developed an increased awareness of its role as a gatekeeper for access (e.g. Stinson, 2004) and a filtering device (e.g. Gates & Vistro-Yu, 2003). Various studies have shown that mathematics is recontextualised differently in different social class settings, tending to distribute more abstract mathematics to the so-called middle-class and a more contextualised mathematics to the so-called working-class (see. e.g. Atweh, Bleicher & Cooper, 1998; Dowling, 1998; Hoadley, 2007; Straehler-Pohl, Fernandes, Gellert, & Figueiras, 2014). All of these studies, either explicitly or by their theoretical underpinnings, highlight the relation between a decontextualised form of mathematics and a pedagogic trajectory towards intellectual labour and a contextualised form of mathematics and a trajectory towards manual labour. The problem I am concerned with in this article is very similar, but still slightly different: In a social context where segregation has advanced to a level where those participating in education (teachers *and* students) are in doubt of considering education as a trajectory towards any form of skilled labour, how does this affect the transmission of mathematical knowledge?

Drawing on the Bernsteinian concepts of the pedagogic device and pedagogic code, I will focus on the role of schools and the transmission of knowledge in the (re)-production of power relations. My perspective on school will be driven by the assumption that schools do not only transmit different amounts of knowledge (or in more fashionable words different levels of graded competence) to different social groups but also different forms of knowledge.

My focus is on mathematical knowledge, as mathematics is considered one of the school subjects in which success and failure have a crucial impact on further vocational and educational opportunities for students. Further, it can stand exemplary for a specialised form of knowledge that is controversially discussed concerning its role as 'powerful knowledge', which distributes power by providing opportunities to produce real outcomes, as opposed to 'knowledge of the powerful', which helps an elite defending their privileges (Young, 2008).

1. Theoretical Perspective

In my analysis, I will investigate what kind of mathematical knowledge is transmitted to students in an underprivileged context and deal with the critical question of the regulative effects that this transmission has for them. The analysis will be based on the theoretical assumption that schools distribute different forms of knowledge to different social groups. This does not simply mean that some disciplines are reserved for several social groups while others are distributed to others. It means that while all students appear to acquire knowledge in one and the same discipline, e.g. mathematics, the form that this disciplinary knowledge takes differs fundamentally. Hence, knowledge transmission and acquisition socialises subjects into different relations to disciplinary knowledge (Bernstein, 1990, 2000).

Schools appear to function as state-run institutions that make people accept their failure (and success) as a result of losing (or winning) in a supposed competition among equals (Pais, 2012) while actually only some have access to the rules of that competition and others don't. As a result, students and teachers often perceive the stratification of success and failure as a result of individual merit rather than as a materialisation of a societal problem (Beck, 1988). Analysing the political dimensions of schooling calls for a theoretical framework that links the micro- and the macro-level of society (Pais & Mesquita). Basil Bernstein's theory of the pedagogic device permits making such links. Additionally, this theory sets out the broader boundaries within which education can have a relatively autonomous impact on society (Apple, 2002).

1.1. The pedagogic device and the organisation of secondary schools in Germany

With the pedagogic device, Bernstein conceives the 'remarkable stability and similarity of education among different political economies' (Apple, 2002:614) in a set of formal rules that are intrinsic to any form of symbolic reproduction, as it occurs for instance in schools. The pedagogic device consists of three different sets of rules that are hierarchically related: distributive rules, recontextualising rules and evaluative rules.

The distributive rules differentiate and stratify different forms of knowledge, consciousness and practice and distribute them to different social groups. In the process of stratification of these forms, they are charged with power:

'At the heart of the 'pedagogic device' is the coding of power whereby the 'thinkable' is discriminated and demarcated, in a fashion which corresponds to the function of 'classification.' In modern, complex societies the contrast between the 'sacred' and the 'profane' is formerly paralleled by the classificatory principles emanating from the higher reaches of the education system. The pedagogic device is a mechanism for the distribution of the 'thinkable' among different social groups, for the identification of what may be thought simultaneously implies who may think it. Social order is thus equivalent to the cosmological order of legitimate categories of consciousness.' (Atkinson, 1985:173)

The distributive rule classifies into what Bernstein calls mundane (allowing to think the thinkable) and esoteric (allowing to think the unthinkable) forms of knowledge. The former regulates the more or less direct relation between a material and an immaterial world. In contrast, the latter regulates the relation itself. That is, esoteric knowledge can produce 'alternative realisations between the material and the immaterial' (Bernstein, 2000:30), it is the 'meeting point of order and disorder' (ibid.) and thus it is the site of the accumulation of power: 'Power relations distribute the unthinkable and the thinkable, and differentiate and stratify groups accomplished by the distributive rules' (p. 31).

In Germany, at the beginning of secondary school, students are streamed into three different types of school at the age of ten (twelve in Berlin and the surrounding area). The further educational and vocational options that these schools give access to articulate a strict hierarchy: Graduating from the upper-stream (the Gymnasium), a student acquires the legal right to study any academic discipline she desires at a university. Graduating from the middle- or the lower-stream, a student is restricted to vocational training. While this stratification of students *de facto* implies the differentiation of social groups, it does this via the distribution of different forms of knowledge.

On a second level, recontextualising rules transform forms of knowledge into pedagogic discourse. 'The recontextualised discourse no longer resembles the original because it has been pedagogised or converted into pedagogic discourse' (Singh, 2002:573). This means that in order to be learnable and teachable, knowledge is dissolved from the ordering principles that have regulated the original discourse it was taken from, and re-organised according to a set of new principles dominated by pedagogic discourse. Mathematics is no longer governed by the application of strictly coherent Aristotelian logic but by the social facts of the logic of transmission and acquisition.

> *Finally the recontextualising principle not only recontextualises the what of pedagogic discourse, what discourse is to become subject and content of pedagogic practice. It also recontextualises the how; that is the theory of instruction. ... The theory of instruction ... contains within itself a model of the learner and of the teacher and of [their] relation. The model of the learner is never wholly utilitarian; it contains ideological elements* (Bernstein, 2000: 34-35)

Ability-streaming in Germany is supposed to create homogeneous learning groups that allow for more effective learning by adapting pedagogic practices to the supposedly different needs of the students in the streams. Thus, differential recontextualisation is an overt aim of German secondary schools. According to the analysis of Rösner (2007:46-58), the model of the learner, which underlies this differential recontextualisation, is based on the differentiation of two kinds of abilities: a 'practical' ability and a 'theoretical' ability. According to this differentiation, upper-stream schools shall provide instructions that are optimised for students who have their supposed strengths in 'abstract thinking', while the lower-stream schools shall optimise instructions for students who have their supposed strengths in 'concrete thinking'.[1] It appears to be an explicit aim of the

[1] Rösner gives the following two examples from provincial laws of education that define the function of the lower-stream schools (educational policy is executed on the level of federal states in Germany):

German school system to stratify trajectories towards intellectual forms of labour by a transmission of esoteric knowledge and a decontextualised orientation towards meanings on the one hand and a trajectory towards manual forms of labour by a transmission of mundane knowledge and a contextualised orientation towards meanings on the other hand.

On a third level, evaluation rules transform pedagogic discourse into pedagogic practice. Thus, it is on this level that the categories which have been created and distributed on the first level and recontextualised on the second level are condensed into a pedagogic practice, in which consciousness is produced. While the distributive rules establish power relations and the recontextualising rules produce principles of how these relations can be controlled, it requires continuous evaluation in pedagogic practice for the reproduction of power relations and principles of control to take place.

Accordingly, it appears most reasonable to find empirical instances of pedagogic practices that create a trajectory towards manual forms of labour in the lower-stream schools. The intention of this article is to investigate whether and how such a trajectory is created in a lower-stream school which operates at the 'Urban Boundaries', the fringes of society where people, despite formally blonging to the first world, cannot benefit of this belonging.

2. The context

As has become evident by relating the pedagogic device to German secondary schools, the lower-stream has an official mandate to prepare students for manual labour. However, firstly, in reality lower-stream schools are not schools that attract students with 'practical' abilities by providing an optimised environment for these abilities, but schools that unite those students who have generally been the least successful in primary schools. Secondly, there is a strong correlation between social class indicators and school-type attendance (Pietsch & Stubbe, 2007). Thirdly, particularly in urban spaces, lower-stream schools appear as the main type of secondary school in deprived areas. Finally, attending the lower-stream school has become a stigma related to social contempt (Wellgraf, 2012). Not seldomly, even low-stream students take this stigma on and describe themselves, partly in irony, partly in resignation, as 'stupid' and 'lazy' and their school as an 'Idiotenschule' (school of idiots) or 'Behindertenschule' (school of retards) (Wellgraf 2012:9). In the public discourse, urban lower-stream schools are hardly considered places that prepare for any form of labour.

In the feeder area of the secondary school, where I have carried out the investigations, 70% of the citizens at the age of fifteen and younger lived on social grants. The migration rate among citizens at the age of up to eighteen was above 80% (Senatsverwaltung für Stadtentwicklung Berlin, 2010). In the public discourse, the borough is often referred to as a ghetto. It is associated with the stigma of the

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'The lower-stream school adresses students, who have a core area of their predispositions, interests and accomplishments in concrete-perceptional thinking and in a practical exposure to things' (school-law of Bavaria, cited in Rösner, 2007, p. 48, translation by the author).

'The lower-stream school transmits a foundational and general knowledge to its students, which is oriented towards realistic situations. Instructions put particular emphasis on practical forms of learning' (school-law of Lower Saxony, cited in Rösner, 2007, p. 48, translation by the author).

'Unterschicht', a lower class that is characterised by unemployment (and often a supposedly contented arrangement with the benefits of social welfare). The 'Unterschicht' is thus perceived as an unproductive class below the so-called working class. According to the teacher, most of the children had unemployed parents and subsisted on social grants (for legal reasons, it was not possible to collect data about the actual students' socio-economic backgrounds).

However, the class of 'Unterschicht' is not only perceived as defined by socio-economic inferiority, but is also often linked to 'certain' ethnicities. With its infrastructure that is dominantly shaped by Turkish and Arabic communities, the observed borough corresponded to the stereotype of a ghetto of the 'Unterschicht'. The businesses, which were visible in the streets, were often run by people with a Turk or Arab ethnicity and are affiliated with labour of low reputation (e.g. grocery stores, kebab-shops or barber-shops) or with labour that is even affiliated with criminal activities (e.g. bars, sport-booking stores or casinos). Adolescents in this district are dominantly ascendants of immigrants in the second or third generation.

While the majority of students at the school usually come from Turkish and Arab ethnic backgrounds, these two groups make up for only six of the fourteen students in the particular class under analysis. Eight students have a Romani ethnic background. Students with a Romani background tend to be confronted with the demand to bridge a particularly big gap between their cultural identities and school culture (Chronaki, 2005; see also Stathopoulou, Gana, Govaris & Appelbaum). While preconceptions about a lack of endeavour to integrate into the German mainstream-culture are prevalent for people of Turk or Arab origin, these stereotypes are even amplified for people with a Romani ethnicity. The teacher's description of her students' social behaviour reflected these stereotypes.

The teacher, who took part in our research project, was selected by the school's principal who introduced her to us as one of the best and most experienced teachers of the school with a good competence in collaborating with the students. At the time of data collection, she has already taught for about thirty years at the school.

3. Methodology

The data reported on in this article originates from a bigger international comparative project (Knipping, Reid, Gellert & Jablonka, 2008). In this project, we have videotaped the mathematics lessons in classrooms during the first three consecutive weeks of secondary school. In this particular case, videotaping took place in September 2009 at the very beginning of the school year in a 7th grade (where students are aged 12 to 14 years). The data corpus for this class contains in total 630 minutes of video material and an extensive open interview (60 minutes) with the teacher. Collecting data at the very beginning of mathematics as a secondary school subject was crucial as we expected that the transition from primary to secondary school was likely to result in a change of what is considered as legitimate and illegitimate knowledge and legitimate and illegitimate behaviour. Further, the students and the teacher were not yet familiar with each other. Hence, we assumed the first weeks as especially important for establishing a particular form of school mathematics discourse that would build the frame for students' developmental trajectories for the acquisition of school mathematics.

The selection of data for the analysis followed several steps. In a first step, I have cursorily analysed the complete data-set in order to find common patterns in the organisation of classroom discourse. Accordingly, I have selected key incidents which act 'as a concrete instance of the workings of abstract principles of social organization' (Erickson, 1977:61), based on a second round of preliminary analysis. Key incidents allow for explicating a theoretical loading the researcher has made about the data in an intuitive judgement (Kroon & Sturm, 2000). For this article, I have chosen two key incidents, of which one includes the very first moment of mathematics learning in the new school and one incident that touches the everyday. In order to analyse teacher-students-interactions from an interactive *and* a structural point of view, I have employed detailed analyses of the classroom discourse[2] in the key-incidents and used them to create vignettes (Erickson, 1986).

> *The vignette is a[n] [...] elaborated, literarily polished version of the account [...] Even the most richly detailed vignette is a reduced account, clearer than life. [...] Thus the vignette does not represent the original event itself, for this is impossible. The vignette is an abstraction; an analytical caricature (of a friendly sort)* (Erickson, 1986:150).

I used the analytic tool of the pedagogic code (Bernstein, 1990, 2000) to interpret the vignettes in relation to the theory of the pedagogic device.

3.1. Analytical tool: Pedagogic codes

'A code is a regulative principle, tacitly acquired, which selects and integrates: (a) relevant meanings, (b) forms of their realization, (c) evoking contexts'. (Bernstein, 1990:14)

According to Bernstein, a code regulates what can be thought and communicated within a certain context. It defines what counts as a context, what meanings are relevant within this context and how they can be realized in communication. A code is therefore the means by which communication produces meaning in relation to a given social context. However, it is 'inseparable from the concept of legitimate and illegitimate communication' (p. 15). Thus, Bernstein makes us aware that establishing one code as legitimate reflexively invokes a devaluation of alternative codes. Thus codes establish power relations and the means to control the power relations. Bernstein operationalizes the pedagogic code by means of classification as a ruler for power and framing as a ruler for control.

3.1.1. Classification as the ruler for power relations

According to Bernstein, what counts as a legitimate context, discourse, knowledge or skill is defined by its boundaries. What something is can only be defined in relation to what it is not. Bernstein therefore uses the term of classification. While always relating 'what it is' to 'what it is not', classifications imply the emergence of power relations:

> *Power relations [...] create boundaries, legitimise boundaries, reproduce boundaries, between different categories of groups, gender, class, race, different categories of discourses, different categories of agents'* (Bernstein, 2000:5).

Bernstein further distinguishes between *internal* and *external classification. External classification* describes a) how strong school knowledge is insulated from everyday knowledge and b) how strong different school subjects are insulated from one another.

[2] I approached 'discourse' from a social semiotic perspective (Halliday & Hasan, 1989).

External classification is coded as strong, where there is a strong insulation, external classification is coded as weak, where the insulation is weak.

The concept of *internal classification* is conceptually undersized in Bernstein's theory, particularly for analysing the internal structure of the school knowledge that is transmitted. Therefor, the concept of classification has been combined with Chevallard's concept of praxeology (Straehler-Pohl & Gellert, 2013, Gellert, Barbé & Espinoza, 2013). Chevallard (1999) considers any school mathematical activity as concerned with the study of a type of problem. In order to solve a problem, a technique has to be developed, which can at least potentially be described, justified or explained. Chevallard distinguishes between two fundamentally different forms of such justification. The first form is called the know-how, which legitimates a technique simply by the correctness of a solution (e.g. through situational adequacy or authority). However, each activity can further be considered as a part of a discursive environment: the know-why. When activities contain this second dimension, this implies the theorisation of techniques, which Chevallard refers to as technology (technique+logos). An activity which systematically develops technologies fixates a coherent, explicit and principled knowledge structure and therefore classifies a discourse from other discourses by producing a specialised form of legitimating knowledge. Hence, strong internal classification goes along with the frequent occurrence of technological moments in the activity.

3.1.2. Framing as the carrier of control relations

> *Framing is about who controls what. [...] Framing refers to the nature of control over: (i) the selection of the communication;(ii) its sequencing (what comes first, what comes second); (iii)its pacing (the rate of expected acquisition); (iv) the criteria; and (v) the control over the social base which makes transmission possible. (Bernstein, 2000:13)*

What counts as pedagogic discourse (classification) can therefore only be defined in practice and is thus a result of how it is practiced (framing). Through different realisations of framing, teachers can thus establish different forms of interaction as legitimate. This can have crucial implications for the positioning of students within the pedagogic practice: Are students given a say in the negotiation of discourse? Do students have opportunities to demand an adaption of pedagogic practice to individual needs? Are students provided with the criteria to legitimate knowledge autonomously? Framing thus bears the potentials for an alternative discourse.

4. Analysis

In order to illustrate the regular pattern behind the vast majority of classroom discourse, I will firstly analyse a vignette that represents how mathematics instruction occurred very regularly in the observed classroom.

4.1. Vignette 1: Revising subtraction (lesson 1)

> *'You surely all know basic operations from primary school. Actually, you all are able to do the addition quite well, also beyond the tens, but what I noticed then is that you very unfortunately forgot how subtraction, division and multiplication worked again.' Therefore, the teacher announces a repetition of the basic operations for the next weeks. 'I will explain subtraction to you once again'. However, the teacher's announcement 'I explain' in turn is substituted firstly by 'I'll do an exercise for practice' and finally by the demand 'so is one of you able to compute 333 minus 18 at the front, for the class?' After two students fail at the blackboard to either carry out the computation or to provide its*

verbalisation the teacher goes through the computation herself: 'Well the first thing is a plusnumber okay?' She writes a plus in front of the number. 'This is a plusnumber from which I shall deduct eighteen, I take the latter number', she points at each digit. 'From eight to three doesn't work right? The three is in fact smaller than the eight. I borrow a ten from the row in front', she writes a small one. Pointing at one digit after the other, she goes on: 'From eight to thirteen are five, okay? I write down the one here because I have borrowed it from the row before. One plus one is two. I add up to three that's one. Don't need to borrow a ten and from zero to three is three, okay? Solution gets underlined twice! ... Any questions?'

4.2. Analysis of the pedagogic code

4.2.1. Classification

The content involved was written subtraction. The teacher gave numbers as an example which were exclusively designed for computation; they were not derived from some apparent real-world example, nor were they in any way related to one of the other school-subjects. It is quite unambiguous that the discourse is characterised by (very) strong external classification.

The teacher assumed that her students 'forgot, how subtraction worked again', this fact is metaphorically indicative for the emphasis on the know-how of the activity: the task of written subtraction calls for remembering a technique, irrespective of understanding or explicating any technological legitimation. This assumption is reinforced during the ongoing episode, especially in the end, when the teacher demonstrated the procedure. Only a very small number of the many procedural steps went along with a reference to a legitimation. Each of these few legitimations were realised implicitly. For example, for the subtraction of ones, she said: 'From eight to three doesn't work right? The three is in fact smaller than the eight. I borrow a ten from the row in front'. The possibility of 'borrowing' tens from the 'row in front' was not reflected as a general possibility made available by the structure of the place value system but as a simple procedure that was legitimised solely by the authority of the teacher. Thus, the know-why about the place value system and the constancy of sums that enabled the teacher's actions remained invisible behind the know-how. The internal classification is (very) weak.

4.2.2. Framing

The teacher unequivocally and overtly exercised the control over the selection: the teacher announced the choice of a 'primary school' topic - this was certainly not a choice made by the adolescent students. Like the selection, the sequencing was fixed. First there was some sort of learning by following a demonstration and verbalisation, then learning by repetition of similar tasks on the worksheet. While the two students who failed at the blackboard challenged that sequence by refusing to verbalise the procedure, the teacher insisted on the sequence and finally carried out the verbalisation herself. Throughout the lesson, it was quite clear what was considered as a legitimate contribution, namely the correct reproduction of the algorithm for written subtraction. The teacher made sure that all students have seen such a correct reproduction of the algorithm at the blackboard before they started working on their own work sheets. Thus, the teacher overtly and explicitly controlled the criteria for evaluation. In contrast, the control over pacing and social order cannot be classified unambiguously. At times the pace was fast and regulated, at others it was left to the students. The teacher partly controlled turns and partly accepted autonomous turns, at times she called for the abidance of social norms, and at times she ignored problematic behaviour.

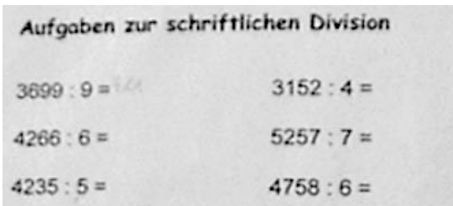
While this vignette was representative for the majority of discourse in that classroom, I would now like to introduce a second vignette, in order to show what happens when the strong external classification is questioned. This second vignette represents one of the very few instances when references to extra-mathematical contexts were made.

4.3. Vignette 2: Sharing Candies (lesson 11)

> *'Now, we did all the operations but one. Which one is lacking? Dragan, read out this word.' 'Division.' 'What does it mean, 'divided'? ... I have a bag of candies with a hundred candies inside and there are ten students in the class. If I want to divide them now, what does it mean?' Yassir answers: 'Everyone gets ten candies.' The teacher now wants to know, how Yassir got his answer. 'Say, I have four candies... I give him two, and I still have two.' Nodding, the teacher acknowledges his answer: 'So you distribute the number of candies to the number of students.' After going through a few examples, where reasonable amounts of candies shall be shared among reasonable amounts of students, the students are left to a work-sheet, with problems similar to 3699:9= . Almost all students struggle with the work sheet. 'What did we say just a minute ago? There I just have a different number.' She writes 3699:9 at the blackboard and together with Dragan demonstrates the written algorithm. Frustrated with Dragan's problems to proceed autonomously, she addresses the class, 'You know what that is? That is primary school, third grade', while an insistent movement of her hand emphasises each word. The teacher then starts from scratch and erases the written algorithm and asks '3699 divided by nine. What does that mean? I have 3699 candies, among how many students shall I distribute them?... And what do I want to know? What do I want to calculate?'*

4.4. Comparative Analysis of vignette 2

Referring back to the precedent revision of basic operations, the aim of the activity was stated as a revision of the 'topic' of division. Thus, the ends of the activity suggest a strong external classification, which already dominated in the previous lessons. However, without having inquired the 'meaning' that the students themselves attach to division, the teacher broke a strong insulation of mathematical and everyday meanings. She claimed that division and sharing candies would not only share a common meaning, but that the meaning of division was derivable straight from the context of sharing candies. Division and sharing candies are depicted as exchangeable for one another. Thus, at this point, the external classification appears considerably weakened. The teacher apparently acknowledged Yassir's answer, which explicitly referenced to the situation of sharing candies instead of a mathematical operation, and thus reinforced the weakening of boundaries. However, in her paraphrase she repeatedly uses the word 'number', and therefore we may already expect an implicit re-strengthening of classification to be on the way. By going through more candy-examples, this re-strengthening was suspended, until it suddenly broke through by means of the introduction of the worksheet (see Fig. 1).


Aufgaben zur schriftlichen Division

3699 : 9 =   3152 : 4 =

4266 : 6 =   5257 : 7 =

4235 : 5 =   4758 : 6 =

Figure 1. Extract from the work-sheet
Note: Elaborated by the author.

However, after witnessing troubles with the worksheet, the teacher came back to the metaphor of sharing candies and demanded an instant dissolution of the boundaries between computation and sharing candies. This weakening of classification for the task of 3699:9 completely mystified the relation between mathematics and the non-mathematical context. Any recontextualisation of meaning from sharing candies to the written division would not improve, but rather hinder the ability to solve the task with the written algorithm. Finally, the weakening of the external classification did not last for long, but - considering the demands of the work sheet - was discarded as abruptly as it was introduced. We can regard the temporarily weak classification as a rather symptomal incident that helps us to identify the excessively marginal meaning of everyday knowledge in the given setting. Everyday knowledge has just a very limited influence on the dominance of strong external classification in this classroom.

Solving tasks such as 3699:9 requires a relatively elaborated technique, such as the written algorithm. However, such a technique has not been practiced in the phase in which the teacher introduced division. Instead, the teacher put an emphasis on the 'meaning' of division. Such an emphasis would imply an emphasis on the know-why as the foundation and thus offer the students the possibility of learning division on the level of technology. However, the teacher legitimised the technique of written division by a reference to candies. This alleged legitimation installed sharing as the technology for written division. The teacher's acknowledgement of Yassir's example suggests that if one was able to distribute a number of candies to a number of students, one understood division. However, moving from rather simple operations like 100 divided by 20 to 3699 divided by nine requires either a proper know-how or else a know-why within a mathematical frame of reference. Introducing sharing candies as the technology that justifies the technique of written division only mystifies the meaning of division. As this apparent technology inevitably collapses when sharing 3699 candies among 9 students, written division remains a technique without a know-why. It acts without any specialised frame of reference. The internal classification is (very) weak.

As a consequence, the evaluation criteria became extremely implicit. The teacher did not only omit to control how to theorise a practice, but also how to execute it. Further, the rapid back and forth concerning the external classification additionally obscured what exactly it was that was expected from the students. The criteria for evaluation, namely to carry out correctly and in right order each step of the algorithm, thus became extremely implicit.

5. Discussion

Given that the students are in seventh grade of a secondary school, the selection of topics is the first aspect that appears to be significant concerning knowledge. Basic operations and its major algorithms belong to the core standards of primary school (grade three and four) as the teacher announced to the students herself. The way the teacher structured her instructions does not give reason for great optimism that students would be enabled to develop an understanding of basic operations, which they apparently had missed out in primary school. That the teacher claimed that students 'forgot' how basic operations 'work' is illustrative of the low expectations of the teacher. These low expectations are manifest in the extremely weak internal classification which effects that mathematics is recontextualised as a set of isolated techniques which need to be memorised by mimicry. Concerning this remote learning of routines, a similar pedagogic practice was in place the whole first week, which was dedicated to carrying out subtractions and the whole second week, which was dedicated to carrying out multiplications. In this way, school mathematics remained a practice without any theory. The students were not provided access to the structuring criteria on which they could develop a competent judgement of their practice. In lack of an internal structure, that is a know-why, students were doomed to be imitators of practices that the teacher inserted as a classroom activity. As there was no transmission of mathematical criteria for the evaluation of students' productions, mathematics does not appear as the aim of pedagogic intervention (cf. Dowling, 1998:27).

What is striking is that the supposed orientation towards a 'practical' ability of the lower-stream school did not seem to have any influence on the strength of the external boundaries of mathematical practice: external classification was consistently strong and the discourse was about nothing else than a 'pure' form of mathematics. Thus, the objective of pedagogic practice in this context appeared to be a clearly institutionalised esoteric mathematics. By its cover, it appeared as a form of specialised discourse. In this classroom, during the rare instances in which students' supposed lives were consulted as a frame of reference, they apparently served to optimize the students' understanding of a strongly institutionalised mathematics as happened in the second vignette. However, as becomes evident in the same vignette, it was obviously a myth that students' supposed everyday can bridge a gap to understanding mathematics: Introducing the context of sharing candies as tantamount and exchangeable for the written algorithm is, with all due respect, ridiculous. Hence, again it appears that the discourse is not really oriented towards mathematics despite the strong external classification. But if the pedagogic practice in this mathematics classroom is not about mathematics, about what is it then?

Respecting the students as active sense makers, we can assert that they were able to see through the inadequacy of the claim that the situation of sharing candies is of help for computing 3699:9. If the candies-analogy is so obviously rather an obstacle than an aid to reaching the mathematical objective of the lesson, namely carrying out the written algorithm for four-digit division problems, what is its pedagogic function?

It appears as if, even though supposedly employed as an instructional tool, candies have a rather regulative function. At the end of the second vignette there appears to be quite an amount of pent-up frustration breaking through in the teacher about the students' supposed inability to carry out the computation in the way she demands. The reference to candies can be read as an illustration of how easy carrying out divisions is supposed to be. Dividing is something that students not only ought to learn, but ought to (already) know. The reference to candies signifies that it actually should not even need systematic teaching to know how to divide.

Finally, in lack of a specialised knowledge structure, pedagogic practice becomes an experience of a cul-de-sac: while there is some knowledge whose acquisition is compulsory, students in this context - in lack of a discourse that would even allow acquisition - are doomed to fail on its acquisition.

6. Conclusion

While classifying the German lower-stream school in the pedagogic device lead to the assumption to find a pedagogic practice that would weakly classify mathematics from students' everyday-experiences, work-life contexts or other school subjects that are relevant for prospective labour, this assumption has been refuted radically. Besides singular instances as exemplified by Vignette 2, mathematics was taught as a strictly esoteric discourse bearing no relation to the world. This points to an irregularity in the process of recontextualisation: there seems to be no 'practicality' in the model of the learner. Against the official objective of the lower stream, students were not prepared for manual labour. It appears that the workings of the pedagogic device, which –as a rule– reproduces the division of labour through selective distributions of orientations to intellectual and manual forms of labour, has been interrupted in this particular context: lower class students are not put off with a mundane form of mathematics, as it is well-known from other studies (Atweh, Bleicher & Cooper, 1998, Dowling, 1998, Hoadley, 2007, Straehler-Pohl, Fernandes, Gellert & Figueiras, 2014). However, the interruption of the pedagogic device did not work in favour of the students. The students became even more subject to a dominant discourse about the 'Unterschicht' which focusses on deficiencies. In the expectation that not only 'practically able' students gather in the low stream, but that it is rather those students who are characterised by 'theoretical dis-ability', the school mathematical discourse had been completely cleared of any internal structure. In this way, school mathematics does not even become a trajectory towards manual labour, but rather makes and marks students as 'disposables' for the divison of labour. In this way, school mathematics becomes a process of social exclusion, as Castells (2000:71) defines it: 'a process by which certain individuals and groups are systematically barred from access to positions that would enable them to an autonomous livelihood within the social standards framed by institutions and values in a given context'.

Now, it would be surely the easiest option to blame the teacher and condemn the reported case as simply a case of bad teaching. Yet, we should bear in mind that she has developed her pedagogic practice in almost 30 years of work in this particular social context and that she is known by the principal and her colleagues as a competent and –measured by the criteria of her immediate environment– even successful teacher. Further, in the interview it had become evident that the teacher authentically refutes a pejorative discourse about her students from an ethical perspective (Straehler-Pohl & Pais, 2014). However, still, it appeared beyond her capacity to refrain from re-enacting this pejorative discourse.

Following this line of thought, it appears sensible to see the teacher's pedagogy as a practice that had been shaped in response to the experiences she made on a daily base. It appears sensible to see her –a professional with more about 30 years of experience– as an expert for the world-life context she is acting in. From this perspective, it is less the individual teacher and more the socio-political structure in which this particular classroom is embedded, that should be held responsible for a pedagogy that tends to social exclusion.

This being said, the teacher's strategy to ignore the official model of the 'practically able' learner and to strengthen the external classification could indicate that the 'practical' forms of mathematics that were available to her, for example in textbooks, proved to be even less utile in the context and that the return to a strictly institutionalised form is a response to this experience.

However, the analysis showed that stripping mathematics completely off any context is counter-productive when it goes without the simultaneous assumption of at least some 'theoretical ability' in the model of the learner: under the preconception of an unreadiness for abstract reasoning, mathematics was transmitted in complete lack of an internal structure. In lack of such structure, pedagogic practice locked students in dependent positions and made them reliant on mimicking the teacher.

The analysis suggests two possible steps out of this cul-de-sac: The first option is transmitting a weakly classified form of mathematics that 'really' takes seriously the real-world contexts it employs. However, mathematics educators that draw on contemporary theory appear pessimistic that a 'real' real-world problem could ever be found (e.g. Dowling, 1998; Davis, 2005; Gerofsky, 2010; Lundin, 2012; Pais, 2013). The second option is transmitting a form of mathematics characterised by relatively strong external classification. However, this would imply seriously assuming 'theoretical ability' for each student and accordingly providing a mathematics with also a strong internal classification. As this second option may appear *unthinkable* in face of the dominant discourse about students from the lowest classes, giving this option a sincere try might be a 'real' attempt to break with this dominant discourse that marginalises the students at the fringes of our societies.

Remarks

Short time after the research has been carried out, the streaming system has been reformed in most of the German federal states (the school-year 2010/11 in Berlin). While the number of streams has been reduced to two and restrictions for permeability between the streams have been lowered, the concept of a streaming system has remained intact.

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